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On The Homogeneous Cubic Equation with Six Unknowns

$$\alpha xy(x+y)+\beta zw(z+w)=(\alpha+\beta)XY(X+Y)$$

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ABSTRACT: *The homogeneous cubic equation with six unknowns represented by the equation $\alpha xy(x+y)+\beta zw(z+w)=(\alpha+\beta)XY(X+Y)$ is analyzed for its patterns of non-zero distinct integral solutions and different methods of integral solutions are illustrated. A few relations between the solutions and the special numbers are presented*

KEYWORDS: *Homogeneous cubic equation, integral solutions*

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NOTATIONS:

$T_{m,n}$ -Polygonal number of rank n with size m

P_n^m - Pyramidal number of rank n with size m

PR_n - Pronic number of rank n

OH_n - Octahedral number of rank n

SO_n -Stella octangular number of rank n

S_n -Star number of rank n

J_n -Jacobsthal number of rank of n

j_n - Jacobsthal-Lucas number of rank n

KY_n -kynea number of rank n

$CP_{n,3}$ - Centered Triangular pyramidal number of rank n

$CP_{n,6}$ - Centered hexagonal pyramidal number of rank n

$F_{4,n,3}$ -Four Dimensional Figurative number of rank n whose generating polygon is a triangle

$F_{4,n,5}$ - Four Dimensional Figurative number of rank n whose generating polygon is a pentagon.

I. INTRODUCTION

The theory of diophantine equations offers a rich variety of fascinating problems [1-3]. Particularly, in [4-9] the cubic equations with 4 unknowns and in [10-11] cubic equations with 5 unknowns are studied for their integral solutions. This communication concerns with an another non-zero cubic equation with six unknowns given by $\alpha xy(x+y)+\beta zw(z+w)=(\alpha+\beta)XY(X+Y)$ **.** Infinitely many non-zero integer triples (x,y,z) satisfying the above equation are obtained. Various interesting properties among the values of x, y and z are presented.

II. METHOD OF ANALYSIS

The diophantine equation representing the cubic equation with six unknowns under consideration is

$$\alpha xy(x+y)+\beta zw(z+w)=(\alpha+\beta)XY(X+Y) \quad (1)$$

Assuming $x=u+p,\ y=u-p,\ z=u+q,\ z=u-q,\ x=u+v,\ x=u-v$ (2)

in (1), it reduces to the equation,

$$\alpha p^2+\beta q^2=(\alpha+\beta)v^2 \quad (3)$$

Again using the linear transformation

$$p = S - \beta T, q = S + \alpha T \tag{4}$$

in (3), it reduces to

$$S^2 + \alpha\beta T^2 = v^2 \tag{5}$$

The above equation (5) is solved through different approaches and thus, one obtains different sets of solutions to (1)

1. CaseI: $\alpha\beta$ is not a square.

1.1 Approach1:

The solution to (5) can be written as

$$S = \alpha\beta a^2 - b^2, T = 2ab, v = \alpha\beta a^2 + b^2 \tag{6}$$

In view of (6), (4) and (2) the integral solution of (1) is obtained as

$$\left.\begin{aligned} x &= u + \alpha\beta a^2 - b^2 - 2\beta ab \\ y &= u - (\alpha\beta a^2 - b^2 - 2\beta ab) \\ z &= u + \alpha\beta a^2 - b^2 + 2\alpha ab \\ w &= u - (\alpha\beta a^2 - b^2 + 2\alpha ab) \\ X &= u + \alpha\beta a^2 + b^2 \\ Y &= u - \alpha\beta a^2 - b^2 \end{aligned}\right\} \tag{7}$$

Properties:

1. $3k[(x+z) - (\alpha\beta - (\alpha - \beta) - 1)(T_{6,a} + 3(OH_a) - 2CP_{a,6})]$ is a nasty number
2. $x(a,a) + w(a,a) + 2(\alpha + \beta)(2T_{3,a} + T_{6,a} - 2T_{4,a}) \equiv 0 \pmod 2$
3. $k + \alpha\beta(6F_{4,a,5} - 2P_a^5 + 2T_{4,a}) - 4\beta T_{3,a} - x(a(a+1),1) = 1$
4. $3(X(a,a) - Y(a,a)) - (1 + \alpha\beta)(S_a + 12CP_{a,3} - 6CP_{a,6} - 1) = 0$
5. $2(z(a,a) - w(a,a)) - (\alpha\beta + 2\alpha - 1)[T_{10,a} + 9(OH)_a - 6CP_{a,6}] = 0$
6. $w(2^{2n}, 2^{2n}) - x(2^{2n}, 2^{2n}) + (\alpha\beta - (\alpha - \beta) - 1) j_{4_{n+1}} + \alpha\beta - \alpha + \beta = 1$.
7. $x(a,a) + X(a,a) - \beta(\alpha - 1)(2T_{3,a} + T_{6,a} - T_{4,a}) = 0$

1.2 Approach2:

Let $v = a^2 + \alpha\beta b^2$ (8)

Now, rewrite (5) as,

$$S^2 + \alpha\beta T^2 = v^2 \times 1 \tag{9}$$

Also 1 can be written as

$$1 = \frac{(\alpha\beta - k^2 + i2k\sqrt{\alpha\beta})(\alpha\beta - k^2 - i2k\sqrt{\alpha\beta})}{(\alpha\beta + k^2)^2} \tag{10}$$

Substituting (10) and (8) in (9) and using the method of factorisation, define

$$(S + i\sqrt{\alpha\beta}T) = \frac{(\alpha\beta - k^2 + i2k\sqrt{\alpha\beta})(a + i\sqrt{\alpha\beta}b)^2}{(\alpha\beta + k^2)} \tag{11}$$

Equating real and imaginary parts in (11) we get

$$\left.\begin{aligned} S &= \frac{1}{(\alpha\beta + k^2)}[(\alpha\beta - k^2)(a^2 - \alpha\beta b^2) - 4\alpha\beta kab] \\ T &= \frac{1}{(\alpha\beta + k^2)}[(\alpha\beta - k^2)2ab + 2k(a^2 - \alpha\beta b^2)] \end{aligned}\right\} \quad (12)$$

Considering (2), (4), (8) & (12) and performing some algebra, the corresponding solutions of (1) are given by

$$\left.\begin{aligned} x &= u + (\alpha\beta + k^2)(f - \beta g) \\ y &= u - (\alpha\beta + k^2)(f - \beta g) \\ z &= u + (\alpha\beta + k^2)(f + \alpha g) \\ w &= u - (\alpha\beta + k^2)(f + \alpha g) \\ X &= u + (\alpha\beta + k^2)^2(A^2 + \alpha\beta B^2) \\ Y &= u - (\alpha\beta + k^2)^2(A^2 + \alpha\beta B^2) \end{aligned}\right\} \quad (13)$$

where

$$\left.\begin{aligned} f &= (\alpha\beta - k^2)(A^2 - \alpha\beta B^2) - 4\alpha\beta kAB \\ g &= (\alpha\beta - k^2)2AB + 2k(A^2 - \alpha\beta B^2) \end{aligned}\right\} \quad (14)$$

Properties:

1. $6k[x(a(a+1), a(a+1)) - \{\alpha\beta + k^2\}\{(\alpha\beta - k^2 - 2k\beta)(1-\alpha\beta) - 2(2\alpha\beta k - \alpha k + k^2)\}$
$\{24F_{4,a,3} - 24P_a^3 + 4T_{3,a}\}]$

   is a nasty number

2. $x(a,a) - y(a,a) + z(a,a) - w(a,a) = 2(\alpha\beta + k^2)[2(\alpha\beta - k^2)(1-\alpha\beta) - 4\alpha\beta k + 2(\alpha - \beta)(\alpha k - k^2)$
$+2k(1-\alpha\beta)][2T_{3,a} - 2CP_{a,6} + SO_a]$

.3. $k^2[w(a,a) + \{\alpha\beta + k^2\}\{(\alpha\beta - k^2 + 2k\alpha)(1-\alpha\beta) - 2(2\alpha\beta k + \alpha^2\beta - \alpha k^2)\}\{2P_a^8 - SO_a\}]$
   is a cubic integer

4. $X(a(a+1), a(a+1)) - Y(a(a+1), a(a+1)) = 2(\alpha\beta + k^2)^2(1+\alpha\beta)(2T_{3,a^2} + 2CP_{a,6})$

1.3 Approach3:

1 can also be written as

$$1 = \frac{(\alpha - \beta + i2\sqrt{\alpha\beta})(\alpha - \beta - i2\sqrt{\alpha\beta})}{(\alpha + \beta)^2}$$

Following the same procedure as in approach2 we get the integral solution of (1) as

$$\left.\begin{aligned} x &= u + (\alpha + \beta)(f_1 - \beta g_1) \\ y &= u - (\alpha + \beta)(f_1 - \beta g_1) \\ z &= u + (\alpha + \beta)(f_1 + \alpha g_1) \\ w &= u - (\alpha + \beta)(f_1 + \alpha g_1) \\ X &= u + (\alpha + \beta)^2(A^2 + \alpha\beta B^2) \\ Y &= u - (\alpha + \beta)^2(A^2 + \alpha\beta B^2) \end{aligned}\right\} \quad (15)$$

Where

$$\left.\begin{aligned} f_1 &= (\alpha-\beta)(A^2-\alpha\beta B^2)-4\alpha\beta AB \\ g_1 &= 2(A^2-\alpha\beta B^2)+2AB(\alpha-\beta) \end{aligned}\right\} \qquad (16)$$

Properties:

1. $6k\{4x(a,a)-(\alpha+\beta)[(\alpha-3\beta)(1-\alpha\beta)-2(\alpha\beta+\beta^2)][T_{10,a}+6T_{3,a}-3T_{4,a}]\}$ is a nasty number
2. $3y(a,a)+(\alpha+\beta)[(3\alpha-\beta)(1-\alpha\beta)-2(\alpha\beta+\beta^2)][T_{8,a}+6T_{4,a}-4T_{5,a}]\equiv 0 \pmod 3$
3. $7z(a,a)-(\alpha+\beta)[(3\alpha-\beta)(1-\alpha\beta)-2(3\alpha\beta-\alpha^2)][2T_{9,a}+10T_{3,a}-5T_{4,a}]\equiv 0 \pmod 7$
4. $k^2\{w(a,a)+(\alpha+\beta)[(3\alpha-\beta)(1-\alpha\beta)-2(3\alpha\beta-\alpha^2)][2P_a^5-CP_{a,6}]\}$ is a cubic integer
5. $X(a(a+1),1)-Y(a(a+1),1)-2(6F_{4,a,5}-2P_a^5)-2\alpha\beta=0$
6. $X(2^{2n},2^{2n})-(1+\alpha\beta)(3J_{4n}+1)=0$

1.4 Approach4:

Rewriting (5) as $v^2-S^2=\alpha\beta T^2$ (17)

Factorisation of the equation (17) gives

$$(v+S)(v-S)=(\alpha T)(\beta T) \qquad (18)$$

Considering (18) and using the method of cross multiplication the non-zero integral solution of (1) are obtained as

$$x=u+m^2\alpha-n^2\beta-2\beta mn$$

$$y=u-(m^2\alpha-n^2\beta-2\beta mn)$$

$$z=u+m^2\alpha-n^2\beta+2\alpha mn$$

$$w=u-(m^2\alpha-n^2\beta+2\alpha mn)$$

$$X=u+m^2\alpha+n^2\beta$$

$$Y=u-m^2\alpha-n^2\beta$$

Properties:

1. $x(2a,a)-y(2a,a)-(4\alpha-5\beta)(6F_{4,a,5}-T^2{}_{4,a})\equiv 0 \pmod 3$
2. $z(a,2a)-w(a,2a)-2\alpha P_a^5+\alpha CP_{a,6}=0$
3. $3(X(a,2a)-Y(a,2a))-(\alpha+4\beta)(S_a-1)\equiv 0 \pmod 6$
4. $3(X(a,a)-x(a,a)+z(a,a))-(5\beta+\alpha)(2T_{5,a}+2T_{3,a}-T_{4,a})\equiv 0 \pmod 3$

1.5 Approach5:

(17) can be written as a set of double equations in five different ways as shown below:

Set1: $v+S=\alpha T, v-S=\beta T$

Set2: $v+S=\alpha\beta, v-S=T^2$

Set3: $v+S=\beta T^2, v-S=\alpha$

Set4: $v+S=\alpha, v-S=\beta T^2$

Set5: $v+S=T^2, v-S=\alpha\beta$

Solving each of the above sets, the corresponding values of v, S and T are given by

Set1: $v=(\alpha+\beta)T_1, S=(\alpha-\beta)T_1, T=2T_1$

Set2: $v=2\alpha_1\beta_1+\alpha_1+\beta_1+2T_1^2+2T_1+1, S=2\alpha_1\beta_1+\alpha_1+\beta_1-2T_1^2-2T_1, T=2T_1+1$

Set3: $v = \beta_1 T^2 + \alpha_1, S = \beta_1 T^2 - \alpha_1$

Set4: $v = 2\beta T_1^2 + \alpha_1, S = \alpha_1 - 2\beta T_1^2, T = 2T_1$

Set5: $v = 2T_1^2 + \alpha_1 \beta, S = 2T_1^2 - \alpha_1 \beta, T = 2T_1$

In view of (4) and (2), the corresponding solutions to (1) obtained from set1 are represented as shown below:

$$x = u + (\alpha - 3\beta)T_1$$
$$y = u - (\alpha - 3\beta)T_1$$
$$z = u + (3\alpha - \beta)T_1$$
$$w = u - (3\alpha - \beta)T_1$$
$$X = u + (\alpha + \beta)T_1$$
$$Y = u - (\alpha + \beta)T_1$$

Properties:

1. $3k[x(a) + z(a) - 4(\alpha - \beta)(2T_{3,a} - 2CP_{a,6} + SO_a)]$ is a nasty number
2. $x(a) - y(a) = 2(5\alpha - 3\beta)[6P_a^3 - 6T_{3,a} + 3(OH_a) - 2CP_{a,3}]$

For simplicity, we exhibit below the integer solutions obtained from sets2 to 5 respectively

Set2:

$$x = u + f(\alpha_1, \beta_1, T_1)$$
$$y = u - f(\alpha_1, \beta_1, T_1)$$
$$z = u + g(\alpha_1, \beta_1, T_1)$$
$$w = u - g(\alpha_1, \beta_1, T_1)$$
$$X = u + (2\alpha_1\beta_1 + \alpha_1 + \beta_1 + 2T_1^2 + 2T_1 + 1)$$
$$Y = u - (2\alpha_1\beta_1 + \alpha_1 + \beta_1 + 2T_1^2 + 2T_1 + 1)$$

Where

$$f(\alpha_1, \beta_1, k) = 2\alpha_1\beta_1 + \alpha_1 + \beta_1 - 2k^2 - 2k - (2\beta_1 + 1)(2k + 1)$$
$$g(\alpha_1, \beta_1, k) = 2\alpha_1\beta_1 + \alpha_1 + \beta_1 - 2k^2 - 2k + (2\alpha_1 + 1)(2k + 1)$$

Set3:

$$x = u + \beta_1 T^2 - \alpha_1 - 2\beta_1 T$$
$$y = u - \beta_1 T^2 + \alpha_1 + 2\beta_1 T$$
$$z = u + \beta_1 T^2 - \alpha_1 + 2\alpha_1 T$$
$$w = u - \beta_1 T^2 + \alpha_1 - 2\alpha_1 T$$
$$X = u + \beta_1 T^2 + \alpha_1$$
$$Y = u - \beta_1 T^2 - \alpha_1$$

Set4:

$$x = u + \alpha_1 - 2\beta T_1^2 - 2\beta T_1$$

$$y = u - (\alpha_1 - 2\beta T_1^2 - 2\beta T)$$

$$z = u + \alpha_1 - 2\beta T_1^2 + 4\alpha_1 T_1$$

$$w = u - (\alpha_1 - 2\beta T_1^2 + 4\alpha_1 T_1)$$

$$X = u + 2\beta T_1^2 + \alpha_1$$

$$Y = u - 2\beta T_1^2 - \alpha_1$$

Set5:

$$x = u + 2T_1^2 - \alpha_1\beta - 2\beta T_1$$

$$y = u - (2T_1^2 - \alpha_1\beta - 2\beta T_1)$$

$$z = u + 2T_1^2 - \alpha_1\beta + 4\alpha_1 T_1$$

$$w = u - (2T_1^2 - \alpha_1\beta + 4\alpha_1 T_1)$$

$$X = u + 2T_1^2 + \alpha_1\beta$$

$$Y = u - 2T_1^2 - \alpha_1\beta$$

2. CaseII:

Choose α and β such that $\alpha\beta$ is a perfect square, say, d^2

(5) is written as $S^2 + (dT)^2 = v^2$ (19)

2.1 Approach6:

The solution of (19) can be written as

$dT = a^2 - b^2, S = 2ab, v = a^2 + b^2$ (20)

Considering (20), (4) & (2) and performing some algebra the integral solution of (1) is obtained as

$$\left.\begin{aligned} x &= u + 2d^2AB - \beta d(A^2 - B^2) \\ y &= u - 2d^2AB + \beta d(A^2 - B^2) \\ z &= u + 2d^2AB + \alpha d(A^2 - B^2) \\ w &= u - 2d^2AB - \alpha d(A^2 - B^2) \\ X &= u + d^2(A^2 + B^2) \\ Y &= u - d^2(A^2 + B^2) \end{aligned}\right\} \quad (21)$$

Properties:

1. $x(2a,a) - y(2a,a) - (4d^2 - 3\beta d)(T_{6,a} + 3T_{4,a} - 2T_{5,a}) = 0$
2. $2(z(2a,a) - w(2a,a)) - (4d^2 - 3\alpha d)(T_{10,a} + 3T_{4,a} - 2T_{5,a}) = 0$
3. $X(2a,a) - Y(2a,a) = d^2(2T_{12,a} + 9T_{4,a} - 2T_{20,a})$
4. $6(x(a,a) + y(a,a) + z(a,a) + w(a,a) + X(a,a) - Y(a,a) - 4k)$ is a nasty number
5. $k^2[X(2^{2n}, 2^{2n}) - d^2(j_{4n+1} + 1)]$ is a cubic integer

2.2 Approach7:

(19) can be written as

$v^2 - (dT)^2 = S^2$ (22)

Writing (22) as a set of double equations as

$v + dT = 1, v - dT = S^2$

and Solving, the corresponding values of v, S *and* T are given by

$v = 2S_1^2 d^2 + 2S_1 d + 1, T = -(2S_1^2 d + 2S_1), S = 2S_1 d + 1$

In view of (4) and (2), the corresponding solutions to (1) are represented as

$x = u + 2S_1 d + 1 + \beta(2S_1^2 d + 2S_1)$

$y = u - 2S_1 d - 1 - \beta(2S_1^2 d + 2S_1)$

$z = u + 2S_1 d + 1 - \alpha(2S_1^2 d + 2S_1)$

$w = u - 2S_1 d - 1 + \alpha(2S_1^2 d + 2S_1)$

$X = u + 2S_1^2 d^2 + 2S_1 d + 1$

$Y = u - 2S_1^2 d^2 - 2S_1 d - 1$

Properties:

1. $x(a) + y(a) = (\alpha + \beta)[d(6F_{4,a,5} - 3CP_{a,6} - T^2_{4,a^2}) + 3T_{4,a} - T_{8,a}]$

2. $X(a) - 2d^2(6P_a^5 - CP_{a,6}) - 2d(2T_{3,a} - T_{4,a}) - k = 1$

2.3 Approach8:

(19) can be rewritten as

$$v^2 - S^2 = (dT)^2 \qquad (23)$$

Writing (23) as a set of double equations in 3 different ways as shown below

Set1: $v + S = T^2, v - S = d^2$

Set2: $v + S = d^2, v - S = T^2$

Set3: $v + S = dT^2, v - S = d$

Solving each of the above sets, the corresponding values of v, S *and* T are given by

Set1: $v = 2(f^2 + e^2) + 2(f + e) + 1, S = 2(e^2 - f^2) + 2(e - f), T = 2e + 1$

Set2: $v = 2(f^2 + e^2), S = 2(f^2 - e^2), T = 2e$

Set3: $v = (T^2 + 1)f, S = (T^2 - 1)f$

In view of (4) and (2), the corresponding solutions to (1) are represented as shown below:

Set1:

$x = u + 2(e^2 - f^2) + 2(e - f) - \beta(2e + 1)$

$y = u - 2(e^2 - f^2) - 2(e - f) + \beta(2e + 1)$

$z = u + 2(e^2 - f^2) + 2(e - f) + \alpha(2e + 1)$

$w = u - 2(e^2 - f^2) - 2(e - f) - \alpha(2e + 1)$

$X = u + 2(f^2 + e^2) + 2(f + e) + 1$

$Y = u - 2(f^2 + e^2) - 2(f + e) - 1$

Set2:

$x = u + 2(e^2 - f^2) - 2\beta e$

$y = u - 2(e^2 - f^2) + 2\beta e$

$z = u + 2(e^2 - f^2) + 2\alpha e$

$w = u - 2(e^2 - f^2) - 2\alpha e$

$X = u + 2(e^2 + f^2)$

$Y = u - 2(e^2 + f^2)$

Set3:

$x = u + (T^2 - 1) f - \beta T$

$y = u - (T^2 - 1) f + \beta T$

$z = u + (T^2 - 1) f + \alpha T$

$w = u - (T^2 - 1) f - \alpha T$

$X = u + (T^2 + 1) f$

$Y = u - (T^2 + 1) f$

Remarkable observation:

If $(x_0, y_0, z_0, w_0, X_0, Y_0)$ be any given integral solution of (1), then the general solution pattern is presented in the matrix form as follows:

Odd ordered solutions:

$$\begin{pmatrix} x_{2n-1} \\ y_{2n-1} \\ z_{2n-1} \\ w_{2n-1} \\ X_{2n-1} \\ Y_{2n-1} \end{pmatrix} = \begin{pmatrix} 1 & 0 & (\alpha+\beta)^{4n-3}(\beta-\alpha) & -2\beta(\alpha+\beta)^{4n-3} \\ 1 & 0 & -(\alpha+\beta)^{4n-3}(\beta-\alpha) & 2\beta(\alpha+\beta)^{4n-3} \\ 1 & 0 & -2\alpha\,(\alpha+\beta)^{4n-3} & (\alpha-\beta)(\alpha+\beta)^{4n-3} \\ 1 & 0 & 2\alpha(\alpha+\beta)^{4n-3} & -(\alpha-\beta)(\alpha+\beta)^{4n-3} \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ p_0 \\ q_0 \end{pmatrix}$$

Even ordered solutions:

$$\begin{pmatrix} x_{2n} \\ y_{2n} \\ z_{2n} \\ w_{2n} \\ X_{2n} \\ Y_{2n} \end{pmatrix} = \begin{pmatrix} 1 & 0 & (\alpha+\beta)^{4n} & 0 \\ 1 & 0 & -(\alpha+\beta)^{4n} & 0 \\ 1 & 0 & 0 & (\alpha+\beta)^{4n} \\ 1 & 0 & 0 & -(\alpha+\beta)^{4n} \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} u_0 \\ v_0 \\ p_0 \\ q_0 \end{pmatrix}$$

The values of x, y, z, w, X and Y in all the above approaches satisfy the following properties:

1. $3(x + y)(z + w + X + Y)$ is a nasty number.

2. $x + y + z + w = 2(X + Y)$
3. $2[x^2 + y^2 + X^2 + Y^2 - (z + w)^2] - (x - y)^2 - (X - Y)^2 = 0$
4. $x^2 - y^2 + z^2 - w^2 + X^2 - Y^2 = 0 \pmod 4$
5. $(x + y)(z + w)(X + Y)$ is a cubical integer
6. $xy - zw + (k - y)^2 - (k - w)^2 = 0$

III. CONCLUSION

Instead of (4), the substitution

$$p = S + \beta T , q = S - \alpha T$$

in (3), reduces it to the same equation (5).Then different solutions can be obtained, using the same patterns

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